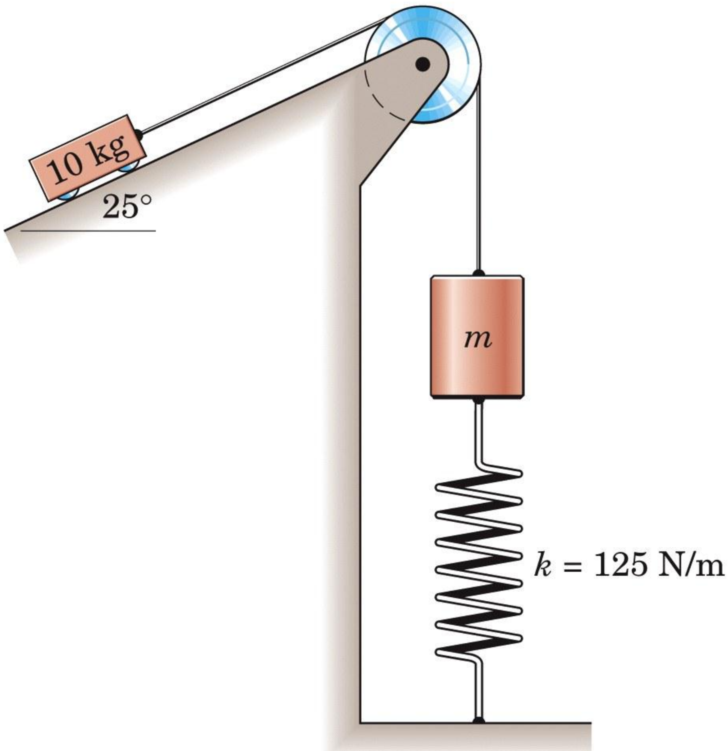
PROBLEM 3/107

The system is released from rest with no slack in the cable and with the spring unstreched. Determine the distance *s* traveled by the 10-kg cart before it comes to rest (a) if *m* approaches zero and (b) if *m* = 2 kg. Assume no mechanical interference.



3/107 Let s be the slant distance down the incline traveled by the 10-kg cart.

(a) $T_1 + U_{1-2} = T_2$

$$0 + m_{10} g s \sin 25° + \tfrac{1}{2}k\left(\cancel{x_1^2} - s^2\right) = 0$$

$$10(9.81)\sin 25° - \tfrac{1}{2}(125)s = 0$$

$\underline{s = 0.663 \text{ m}}$

(b) $T_1 + U_{1-2} = T_2$

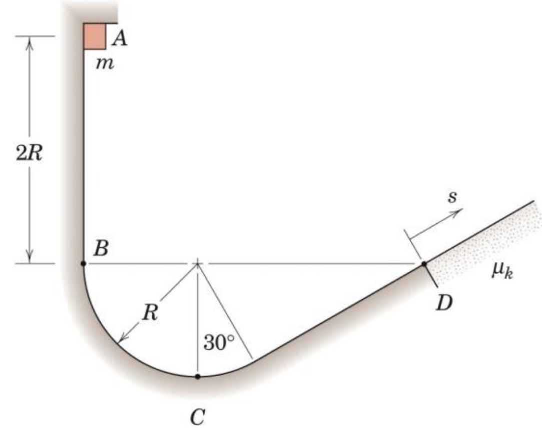
$$0 + m_{10} g s \sin 25° - m_2 g s - \tfrac{1}{2}ks^2 = 0$$

$$10(9.81)\sin 25° - 2(9.81) - \tfrac{1}{2}(125)s = 0$$

$\underline{s = 0.349 \text{ m}}$

PROBLEM 3/125

The small slider of mass m is released from rest while in position A and then slides along the vertical-plane track. The track is smooth from A to D and rough (coefficient of kinetic friction μ_k) from point D on. Determine (a) the normal force N_B exerted by the track on the slider just after it passes point B, (b) the normal force N_C exerted by the track on the slider as it passes the bottom point C, and (c) the distance s traveled along the incline past point D before the slider stops.



3/125 $T_A + U_{A-B} = T_B$: $0 + 2mgR = \frac{1}{2}mv_B^2$, $v_B^2 = 4gR$

(a) mg, N_B, n, t

$\Sigma F_n = ma_n$: $N_B = m\,\frac{4gR}{R} = \underline{4mg}$

(b) $T_A + U_{A-C} = T_C$: $0 + 3mgR = \frac{1}{2}mv_C^2$, $v_C^2 = 6gR$

n, mg, t, N_C

$\Sigma F_n = ma_n$: $N_C - mg = m\,\frac{6gR}{R}$

$\underline{N_C = 7mg}$

(c) Call stopping point E :

$T_A + U_{A-E} = T_E$

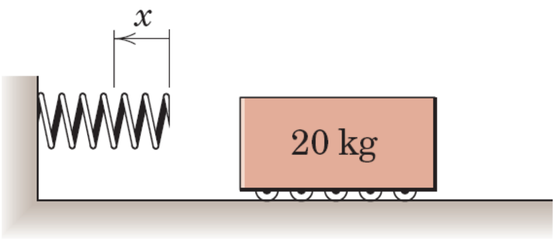
$0 + 2mgR - mg\left(\frac{1}{2}s\right) - \mu_k \frac{\sqrt{3}}{2}mgs = 0$

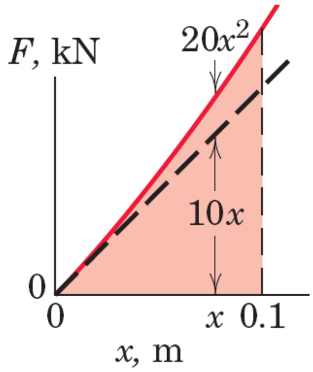
$$\underline{s = \frac{4R}{1 + \mu_k\sqrt{3}}}$$

(Note: Normal force on incline is
$N = mg\cos 30° = \frac{\sqrt{3}}{2}mg$)

PROBLEM 3/133

Calculate the horizontal velocity v with which the 20-kg carriage must strike the spring in order to compress it a maximum of 100 mm. The spring is known as «hardening» spring, since its stiffness increases with deflection as shown in the accompanying graph.





3/133

$$U = \Delta T$$

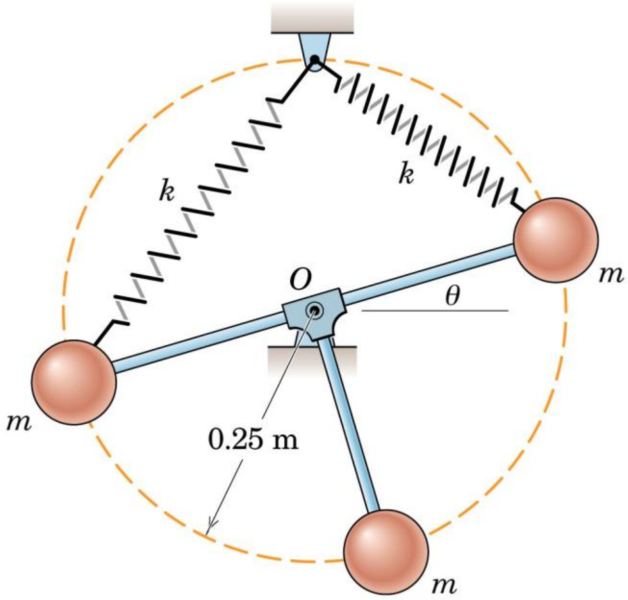
$$-\int_0^{0.1} (20x^2 + 10x)\, 10^3\, dx = \frac{1}{2}(20)(0 - v^2)$$

$$-\left[\frac{20}{3}x^3 + 5x^2\right]_0^{0.1} (10^3) = -10v^2$$

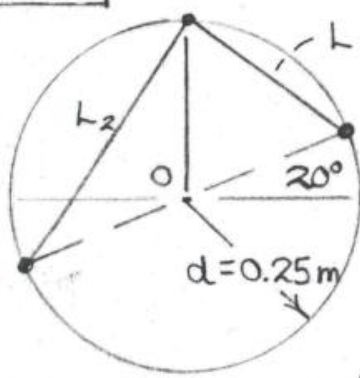
$$\underline{v = 2.38 \text{ m/s}}$$

PROBLEM 3/151

The two springs, each of stiffness k=1.2 kN/m, are of equal length and undeformed when θ=0. If the mechanism is released from rest in the position θ=20^{0}, determine its angular velocity $\dot{\theta}$ when θ=0. The mass m of each sphere is 3 kg. Treat the spheres as particles and neglect the masses of the light rods and springs.



3/151



$$L_1 = 2d\sin(90°-20°)/2 = 0.287\text{ m}$$
$$\delta_1 = 0.25\sqrt{2} - L_1 = 0.0668\text{ m}$$
$$L_2 = 2d\sin\left(\frac{90°+20°}{2}\right) = 0.410\text{ m}$$
$$\delta_2 = L_2 - 0.25\sqrt{2} = 0.0560\text{ m}$$

We may ignore the equal and opposite potential energy changes associated with two of the masses.

$T_1 + V_1 = T_2 + V_2$, datum at O.

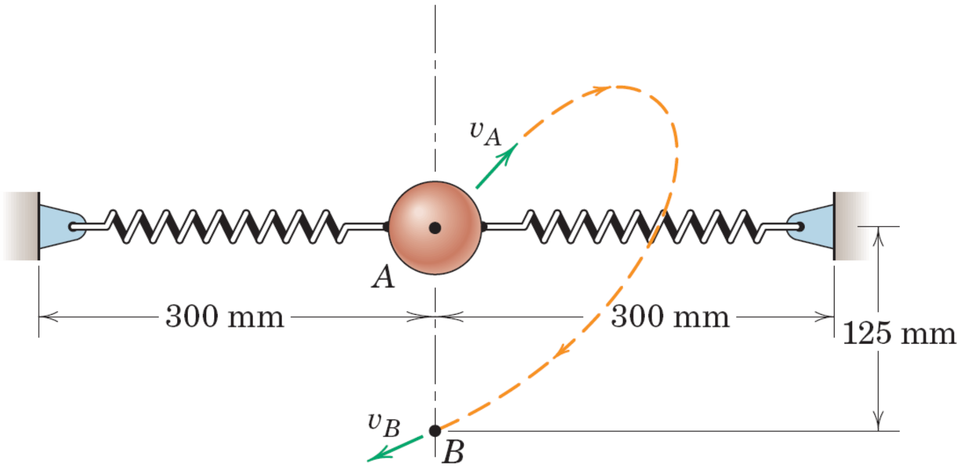
$$0 - mgd\cos 20° + \tfrac{1}{2}k\delta_1^2 + \tfrac{1}{2}k\delta_2^2 = 3\left(\tfrac{1}{2}md^2\dot{\theta}^2\right) - mgd$$

$$0 - 3(9.81)(0.25)\cos 20° + \tfrac{1}{2}1200(0.0668)^2$$
$$+ \tfrac{1}{2}1200(0.0560)^2 = \tfrac{3}{2}3(0.25)^2\dot{\theta}^2 - 3(9.81)(0.25)$$

Solving, $\underline{\dot{\theta} = 4.22\text{ rad/s}}$

PROBLEM 3/153

The 1.5-kg ball is given an initial velocity v_A=*2.5 m/s* in the vertical plane at position A, where the two horizontal attached springs are unstretched. The ball follows the dashed path shown and crosses point B, which is 125 mm directly below A. Calculate the velocity v_B of the ball at B. Each spring has a stiffness of 1800 N/m.



3/153 The system is conservative so

$\Delta T + \Delta V_e + \Delta V_g = 0$, Spring stretch is

$$\sqrt{(300)^2 + (125)^2} - 300 = 25 \text{ mm}$$

$$\Delta T = \frac{1}{2} 1.5 (v_B^2 - [2.5]^2) = 0.75 v_B^2 - 4.688 \text{ J}$$

$$\Delta V_e = 2\left[\frac{1}{2} 1800 (25 \times 10^{-3})^2\right] = 1.125 \text{ J}$$

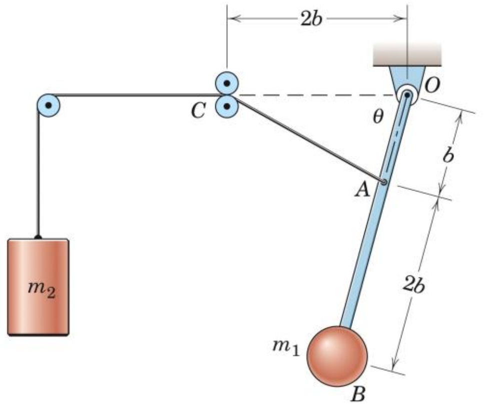
$$\Delta V_g = 1.5(9.81)(-0.125) = -1.839 \text{ J}$$

Thus $0.75 v_B^2 - 4.688 + 1.125 - 1.839 = 0$

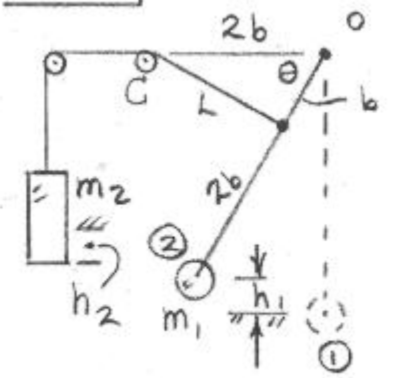
$$v_B^2 = 7.202, \quad \underline{v_B = 2.68 \text{ m/s}}$$

PROBLEM 3/171

The system is released from rest with the angle θ= 90^0. Determine $\dot{\theta}$ when θ reaches 60^0. Use the values m_1=1 kg, m_2=1.25 kg, and b=0.40 m. Neglect friction and the mass of bar OB, and treat the body B as a particle.



3/171



$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

Datums at ① ($\theta = 90°$)

$$0 + 0 + 0 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + m_1 g h_1 - m_2 g h_2 \quad *$$

$$v_1 = 3b\dot{\theta} \quad , \quad h_1 = 3b(1 - \sin\theta)$$

$$L^2 = b^2 + (2b)^2 - 2(b)(2b)\cos\theta$$

$$= 5b^2 - 4b^2\cos\theta = b^2(5 - 4\cos\theta)$$

$$2L\dot{L} = 4b^2 \sin\theta\,\dot{\theta} \quad , \quad \dot{L} = \frac{2b^2 \sin\theta\,\dot{\theta}}{L} = -v_2$$

$$h_2 = L_{\theta=90°} - L = \sqrt{5}\,b - b\sqrt{5 - 4\cos\theta}$$

$$= b\left[\sqrt{5} - \sqrt{5 - 4\cos\theta}\right]$$

$$*: 0 = \frac{1}{2} m_1 (3b\dot{\theta})^2 + \frac{1}{2} m_2 \left(\frac{2b^2 \sin\theta\,\dot{\theta}}{b\sqrt{5 - 4\cos\theta}}\right)^2$$

$$+ m_1 g\, 3b(1 - \sin\theta) - m_2 g b\left[\sqrt{5} - \sqrt{5 - 4\cos\theta}\right]$$

Solve for $\dot{\theta}$:

$$\dot{\theta} = -\left\{\frac{g\left[-3m_1(1 - \sin\theta) + m_2\left(\sqrt{5} - \sqrt{5 - 4\cos\theta}\right)\right]}{b\left[\frac{9}{2} m_1 + 2m_2 \frac{\sin^2\theta}{5 - 4\cos\theta}\right]}\right\}^{1/2}$$

Numbers: $\underline{\dot{\theta} = -1.045 \text{ rad/s}}$ at $\theta = 60°$

(θ is decreasing)